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Multiple Gluon Effects in Fermion–(Anti)Fermion Scattering at SSC/LHC Energies*

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ABSTRACT

We extend the methods of Yennie, Frautschi and Suura (YFS) to compute, via Monte Carlo methods, the effects of multiple gluon emission in the processes $q + {}^{(}\bar{q}^{)\prime} \rightarrow q'' + {}^{(}\bar{q}^{)\prime\prime\prime} + n(G)$, where G is a soft gluon. We show explicitly that the infrared singularities in the respective simulations are canceled to all orders in α_s. Some discussion of this result from the standpoint of confinement is given. More importantly, we present, for the first time ever, sample numerical Monte Carlo data on multiple soft gluon emission in the rigorously extended YFS framework. We find that such soft gluon effects must be taken into account for precise SSC/LHC physics simulations.

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1 Introduction

While the SSC has been cancelled and is being re-evaluated, the LHC planning stages continue to gain momentum, so that it still becomes more and more necessary to prepare for the physics exploration which the LHC, at the least, will provide. The primary issue in this exploration is the comparison between predictions from the standard $SU_{2L} \otimes U_1 \otimes SU_3^C$ theory and its possible extensions, and what will be observed by the GEM and SDC collaborations, with the discovery of the Higgs particle or whatever it represents as of course a primary goal of such comparisons. Thus, it is important to know the theoretical predictions as accurately as is necessary to exploit the expected detector performances of GEM and SDC to the fullest extent. In particular, higher order radiative corrections to these predictions (due to multiple photon and multiple gluon effects) are in fact essential to obtain the proper precision on the signal and background processes in the respective SSC/LHC environment. In Refs. [1], we have developed Yennie–Frautschi–Suura [2] (YFS) multiphoton Monte Carlo event generators for calculating $n\gamma$ effects in SSC/LHC processes. In what follows, we now develop the extension of our own YFS Monte Carlo–based higher order radiative correction methods to multiple gluon effects in SSC/LHC processes. Such a calculation of $n(G)$ effects has not appeared elsewhere, where we use G to denote a gluon.

More precisely, we want to use the fact that, by the uncertainty principle, infinite–wavelength gluons should not affect the motion of quarks and gluons inside a proton, whose radius is ~ 1 fm. Thus, we have recently realized [3] this physical requirement in perturbation theory by showing that in our prototypical processes $q + {}^{(}\bar{q}^{)\prime} \rightarrow q'' + {}^{(}\bar{q}^{)\prime\prime\prime} + (G)$, the infrared singularities cancel at $O(\alpha_s)$, just as they cancel in the analogous QED process $f + f' \rightarrow f + f' + \gamma$. We then are able to define the QCD analogues of the famous YFS infrared functions [2] B and $\tilde{\mathrm{B}}$ which describe the virtual and real infrared singularities in QED processes to all orders in α. These functions, $B_{\rm QCD}$ and $\tilde{B}_{\rm QCD}$, then describe the infrared singularities in QCD in the processes $q + {}^{(}\bar{q}^{)\prime} \rightarrow q'' + {}^{(}\bar{q}^{)\prime\prime\prime} + n(G)$ to all orders in α_S. The sum $B_{\rm QCD} + \tilde{B}_{\rm QCD}$ is infrared finite and allows us to extend our YFS Monte Carlo methods, widely used in SLC/LEP physics for higher order QED radiative effects, to the analogous Monte Carlo methods in QCD processes for SSC/LHC

physics objectives. We call the resulting programs SSCYFSG and SSCBHLG, for they are the QCD extensions of the QED $n\gamma$ Monte Carlos SSCYFS2 and SSCBHL. Here we recall that SSCYFS2 was already described in Ref. [1] and SSCBHL is the analogous SSC extension of the YFS Monte Carlo BHLUMI 2.01 [4] for the SLC/LEP luminosity process $e^+e^- \to e^+e^- + n\gamma$.

In what follows, we shall present sample Monte Carlo data for the process $q + {}^{(}\bar{q}^{)\prime} \to q'' + {}^{(}\bar{q}^{)\prime\prime\prime} + n(G)$ from the event generator SSCBHLG. Analogous results from SSCYFSG will appear elsewhere [5]. The latter Monte Carlo only simulates initial–state gluon radiation, whereas SSCBHLG simulates initial–state and final–state n–gluon radiation, as well as the respective initial–final–state interference effects. Further, we emphasize that (unlike BHLUMI 2.01) SSCBHLG is not restricted to small scattering angles. Note finally that SSCYFSG is an initial–state restriction of SSCBHLG and hence is useful for cross–checking our work.

Our work is organized as follows. In the next Section, we derive the extension of the YFS exponentiation to n–gluon emission and the attendant extension of SSCYFS2 and SSCBHL. In Section III, we present sample Monte Carlo data for n–gluon emission for the SDC and GEM acceptances. Section IV contains some concluding remarks.

2 Infrared Singularity Cancellation in QCD

In this Section, we derive the analogue of the YFS infrared functions B and $\tilde{\mathrm{B}}$ for QCD, $B_{\rm QCD}$ and $\tilde{B}_{\rm QCD}$, for the process $q + {}^{(}\bar{q}^{)\prime} \to q'' + {}^{(}\bar{q}^{)\prime\prime\prime} + (G)$. We will focus on the case of most interest to us, in which a gluon is exchanged in the t–channel and we require that the exchanged gluon carries large momentum transfer so that the outgoing quark (anti–quark) is in the $|\eta| \leq 2.8$ acceptance of the SDC (and GEM). This situation is depicted in Fig. 1. We remark that if the relevant exchange is a color singlet exchange (such as W or Z^0 exchange), we can obtain the corresponding formulas for $B_{\rm QCD}$ and $\tilde{B}_{\rm QCD}$ by dropping the terms in the color–exchange case which arise from non–commutativity of the respective QCD λ^a matrices in the relevant quark representation.

Specifically, following the kinematics in Fig. 1a, we note that, for the SDC and GEM

acceptance, we always have $-Q^2 = -(q_1 - q_2)^2 \gg \Lambda^2_{\rm QCD}$, so that perturbative QCD methods are applicable. What we wish to do is to extract the infrared–singular part of the $O(\alpha_s)$ amplitude in Fig. 1b, in complete analogy with the YFS [2] extraction of the infrared–singular portion of the analogous amplitude in QED via the famous YFS virtual infrared function B. To this end, we first note that, by explicit calculation, the graphs (v)-(vii) are not infrared–divergent (we compute the gluon vacuum polarization to order α_s as well), so that the infrared–singular part of Fig. 1b is given by just the same graphs as the infrared–singular part of the analogous process in QED. We follow the YFS methods in Ref. [2] and find the infrared–singular part of Fig. 1b to be given by

$$\mathcal{M}_{\rm v,IR} = \alpha_s B_{\rm QCD}\mathcal{M}_{8B} + \alpha_s \bar{B}_{\rm QCD}\mathcal{M}_{0B} \quad , \tag{1}$$

where $\mathcal{M}_{8B}$ is the respective Born amplitude in Fig. 1a and where

$$\begin{aligned} B_{\rm QCD} = \frac{i}{(8\pi^3)}\int \frac{d^4k}{(k^2-\lambda^2+i\epsilon)}\Bigg[& C_F\left(\frac{2p_1+k}{k^2+2k\cdot p_1+i\epsilon}+\frac{2p_2-k}{k^2-2k\cdot p_2+i\epsilon}\right)^2 \\ +\Delta C_s\frac{2(2p_1+k)\cdot(2p_2-k)}{(k^2+2k\cdot p_1+i\epsilon)(k^2-2k\cdot p_2+i\epsilon)} & +C_F\left(\frac{2q_1+k}{k^2+2k\cdot q_1+i\epsilon}-\frac{2q_2-k}{k^2-2k\cdot q_2+i\epsilon}\right)^2 \\ +\Delta C_s\frac{2(2q_1+k)\cdot(2q_2-k)}{(k^2+2k\cdot q_1+i\epsilon)(k^2-2k\cdot q_2+i\epsilon)} & +C_F\left(\frac{2p_2+k}{k^2+2k\cdot p_2+i\epsilon}-\frac{2q_2+k}{k^2+2k\cdot q_2+i\epsilon}\right)^2 \\ +\Delta C_t\frac{2(2q_2+k)\cdot(2p_2+k)}{(k^2+2k\cdot q_2+i\epsilon)(k^2+2k\cdot p_2+i\epsilon)} & +C_F\left(\frac{2p_1+k}{k^2+2k\cdot p_1+i\epsilon}-\frac{2q_1+k}{k^2+2k\cdot q_1+i\epsilon}\right)^2 \\ +\Delta C_t\frac{2(2q_1+k)\cdot(2p_1+k)}{(k^2+2k\cdot q_1+i\epsilon)(k^2+2k\cdot p_1+i\epsilon)} & -C_F\left(\frac{2p_1+k}{k^2+2k\cdot p_1+i\epsilon}-\frac{2q_2+k}{k^2+2k\cdot q_2+i\epsilon}\right)^2 \\ +\Delta C_u\frac{2(2p_1+k)\cdot(2q_2+k)}{(k^2+2k\cdot p_1+i\epsilon)(k^2+2k\cdot q_2+i\epsilon)} & -C_F\left(\frac{2q_1+k}{k^2+2k\cdot q_1+i\epsilon}-\frac{2p_2+k}{k^2+2k\cdot p_2+i\epsilon}\right)^2 \\ & +\Delta C_u\frac{2(2q_1+k)\cdot(2p_2+k)}{(k^2+2k\cdot q_1+i\epsilon)(k^2+2k\cdot p_2+i\epsilon)} \end{aligned} \tag{2}$$

with $C_F = 4/3$ =quadratic Casimir invariant of the quark color representation, λ equal to the standard infrared regulator mass (Ref. [6]) and

$$\begin{aligned} \Delta C_s &= \begin{cases} -1, & qq' \text{ incoming} \\ -1/6, & q\bar{q}' \text{ incoming} \end{cases} , \quad \Delta C_t = -3/2, \text{ and} \\ \Delta C_u &= \begin{cases} -5/2, & qq' \text{ incoming} \\ -5/3, & q\bar{q}' \text{ incoming} \end{cases} . \end{aligned} \tag{3}$$

Thus, we see that the non–Abelian nature of QCD causes $\Delta C_j \neq 0$.

The additional term in (1), which arises from color algebra, consists of what we call the "singlet" exchange Born amplitude which is obtained from Fig. 1a by substituting λ^0 for the color matrices λ^a, where $\lambda^0 \equiv I/\sqrt{6}$. The corresponding value of $\bar{B}_{\rm QCD}$ in (1) is obtained from (2) by setting $C_F \equiv 0$ and $\Delta C_t = 0$ in (2) and by setting

$$\Delta C_u = \Delta C_s = \begin{cases} -C_F, & qq' \text{ incoming} \\ C_F, & q\bar{q}' \text{ incoming} \end{cases} \tag{4}$$

in (2). We note that, to order α_s, the "singlet" amplitude does not contribute to our cross section for the process in Fig. 1.

Turning now to the soft real emission process in Fig. 1b, we again follow the methods of YFS in Ref. [2] and extract the infrared–divergent part of the real gluon bremsstrahlung as

$$d\sigma^{B1}_{soft} = d\sigma_0(2\alpha_s \tilde{B}_{\rm QCD}) \tag{5}$$

where the real QCD infrared function is

$$2\alpha_s \tilde{B}_{\rm QCD} = \int^{k \le K_{max}} \frac{d^3k}{k_0} \tilde{S}_{\rm QCD}(k) \tag{6}$$

with

$$\begin{aligned} \tilde{S}_{\rm QCD}(k) = -\frac{\alpha_s}{4\pi}\Bigg\{ & C_F\left(\frac{p_1}{p_1\cdot k}-\frac{q_1}{q_1\cdot k}\right)^2 + \Delta C_t \frac{2p_1\cdot q_1}{k\cdot p_1 k\cdot q_1} + C_F\left(\frac{p_2}{p_2\cdot k}-\frac{q_2}{q_2\cdot k}\right)^2 + \Delta C_t \frac{2p_2\cdot q_2}{k\cdot p_2 k\cdot q_2} \\ & + C_F\left(\frac{p_1}{p_1\cdot k}-\frac{p_2}{p_2\cdot k}\right)^2 - \Delta C_s \frac{2p_1\cdot p_2}{k\cdot p_1 k\cdot p_2} + C_F\left(\frac{q_1}{q_1\cdot k}-\frac{q_2}{q_2\cdot k}\right)^2 - \Delta C_s \frac{2q_1\cdot q_2}{k\cdot q_1 k\cdot q_2} \\ & - C_F\left(\frac{q_1}{q_1\cdot k}-\frac{p_2}{p_2\cdot k}\right)^2 + \Delta C_u \frac{2q_1\cdot p_2}{k\cdot q_1 k\cdot p_2} - C_F\left(\frac{q_2}{q_2\cdot k}-\frac{p_1}{p_1\cdot k}\right)^2 + \Delta C_u \frac{2q_2\cdot p_1}{k\cdot q_2 k\cdot p_1}\Bigg\}, \end{aligned} \tag{7}$$

where K_{max} corresponds to the relevant gluon jet detector resolution energy ($\sim$ 3 GeV at the SSC/LHC) and $k_0 \equiv \sqrt{\vec{k}^2 + \lambda^2}$ so that we regulate the infrared singularities in (7) in complete analogy with our gluon mass regulator in (2). (Note that graph (v) in Fig. 1c is not infrared–divergent.) Here, $d\sigma_0$ is the respective Born differential cross section. The results (1) to (7) represent the complete infrared (IR) structure of QCD for the cross section for

$q + {}^{(}\bar{q}^{)\prime} \to q'' + {}^{(}\bar{q}^{)'''} + (G)$ to order α_s. The result (7) is consistent with the pioneering work of Berends and Giele in Ref. [7]. Following the YFS theory, we see that the criterion for the cancellation of the $O(\alpha_s)$ and IR singularities in the cross section for our scattering process in Fig. 1 is that

$$\mathrm{SUM}_{\mathrm{IR}}(\mathrm{QCD}) \equiv 2\alpha_s \tilde{B}_{\mathrm{QCD}} + 2\alpha_s \mathrm{Re} B_{\mathrm{QCD}} \tag{8}$$

should be independent of λ. Using the well–known results [2],[8] for the integrals in (2) and (7) (notice that these are just the same integrals that arise in the YFS analysis of QED with different color–based weights) we see that, indeed, λ does cancel out of (8). We are left with the fundamental result (for, e.g., $m_q = m_{q'} = m$)

$$\mathrm{SUM}_{\mathrm{IR}}(\mathrm{QCD}) = \frac{\alpha_s}{\pi} \sum_{A=\{s,t,u,s',t',u'\}} (-1)^{\rho(A)} (C_F B_{tot}(A) + \Delta C_A B'_{tot}(A)) \tag{9}$$

where

$$B_{tot}(A) = \log(2K_{max}/\sqrt{|A|})^2 (\ln(|A|/m^2) - 1) + \frac{1}{2}\ln(|A|/m^2) - 1 - \pi^2/6 + \theta(A)\pi^2/2 \quad ,$$

$$B'_{tot}(A) = \log(2K_{max}/\sqrt{|A|})^2 + \frac{1}{2}\ln(|A|/m^2) - \pi^2/6 + \theta(A)\pi^2/2 \tag{10}$$

and

$$\rho(A) = \begin{cases} 0, & A = s, s', \quad t, t' \\ 1, & A = u, u' \end{cases} \quad . \tag{11}$$

The results (9) and (10) are then the fundamental results of our analysis.

Indeed, repeating the arguments in Ref. [2] and/or using the factorization results in Ref. [7], we see that by virtue of (9), we may compute the hard gluon radiation residuals at $O(\alpha_s)$ as

$$\bar{\beta}_0 = d\sigma^{1-loop} - 2\alpha_s \mathrm{Re} B_{\mathrm{QCD}} d\sigma_0 \tag{12}$$

and

$$\bar{\beta}_1 = d\sigma^{B1} - \tilde{S}_{\mathrm{QCD}} d\sigma_0 \tag{13}$$

in the exponentiated formula

$$d\sigma_{exp} = exp[\mathrm{SUM}_{\mathrm{IR}}(\mathrm{QCD})] \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{+iy(p_1+p_2-q_1-q_2-\sum_j k_j)+D} \bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3 q_1 d^3 q_2}{q_1^0 q_2^0} \tag{14}$$

where

$$D = \int \frac{d^3k}{k_0} \tilde{S}_{\rm QCD} \left[e^{-iy\cdot k} - \theta(K_{max} - |\vec{k}|) \right] \quad . \tag{15}$$

Here, we use the virtual 1–loop correction

$$d\sigma^{1-loop} = d\sigma_0 \delta_{vir} \, , \tag{16}$$

where

$$\begin{aligned} \delta_{vir} = \tfrac{2\alpha_s}{\pi} \quad & \Big[(C_F - \tfrac{1}{2}C_A) \Big\{ \ln\tfrac{|t|}{m^2} \Big(2 - \ln\tfrac{|t|}{\lambda^2} + \tfrac{1}{2}\ln\tfrac{|t|}{m^2} \Big) \\ & \qquad + \ln\tfrac{s}{m^2} \Big(2\ln\tfrac{s}{\lambda^2} - \ln\tfrac{s}{m^2} \Big) + \tfrac{tu}{s^2+u^2} \ln\tfrac{s}{|t|} \Big(\tfrac{1}{2}(s/u - 1)\ln\tfrac{s}{|t|} - 1 \Big) \Big\} \\ & -(C_F - \tfrac{1}{4}C_A) \Big\{ \ln\tfrac{|u|}{m^2} \Big(2\ln\tfrac{|u|}{\lambda^2} - \ln\tfrac{|u|}{m^2} \Big) + \tfrac{st}{s^2+u^2} \ln\tfrac{u}{t} \Big(\tfrac{1}{2}(1 - u/s) \; \ln\tfrac{u}{t} - 1 \Big) \Big\} \\ & + C_F \ln\tfrac{|t|}{\lambda^2} + C_A \Big(\ln\tfrac{|t|}{m^2} + 1 \Big) + \tfrac{31}{36}C_A - \tfrac{5}{9}C_F + \tfrac{\pi^2}{6}C_F \Big] \end{aligned} \tag{17}$$

The respective soft real gluon bremsstrahlung cross is given by (5) with

$$\begin{aligned} 2\alpha_s \tilde{B}_{QCD} = \tfrac{2\alpha_s}{\pi} \quad & \Big[(C_F - \tfrac{1}{2}C_A) \Big\{ \ln\tfrac{4K_{max}^2}{\lambda^2} + \ln\tfrac{|t|}{m^2} \Big(1 + \ln\tfrac{4K_{max}^2}{\lambda^2} - \tfrac{1}{2}\ln\tfrac{|t|}{m^2} \Big) \\ & \qquad -2\ln\tfrac{s}{m^2} \Big(1 + \ln\tfrac{4K_{max}^2}{\lambda^2} - \tfrac{1}{2}\ln\tfrac{s}{m^2} \Big) \Big\} \\ & + (C_F - \tfrac{1}{4}C_A) \Big\{ -2\ln\tfrac{4K_{max}^2}{\lambda^2} + 2\ln\tfrac{|u|}{m^2} \Big(1 + \ln\tfrac{4K_{max}^2}{\lambda^2} - \tfrac{1}{2}\ln\tfrac{|u|}{m^2} \Big) - \tfrac{\pi^2}{3} \Big\} \Big] \end{aligned} \tag{18}$$

. It allows $\bar{\beta}_0$ to be identified as $d\sigma_{soft}^{\mathcal{O}(\alpha_s)} - 2\alpha_s(\tilde{B}_{QCD} + B_{QCD})d\sigma_0$, where $d\sigma_{soft}^{\mathcal{O}(\alpha_s)} = d\sigma^{1-loop} + d\sigma_{soft}^{B1}$, for example. The above expressions are for incoming $q\bar{q}$ and need to be modified accordingly for qq interactions. The hard gluon residual $\bar{\beta}_1$ makes a vanishing contribution in the soft gluon limit so that it will be presented elsewhere [5]; for, here we focus on the soft gluon effects in (14) so that we will not need $\bar{\beta}_1$ in the current discussion.

The results (12) and (16) – (18) are now realized in (14) via Monte Carlo methods by replacing the QED forms of $\bar{\beta}_n$, $\mathrm{SUM_{IR}}$ as described in Refs. [1] for our SSCYFS2 and SSCBHL event generators by the respective QCD forms. The resulting event generators are SSCYFSG and SSCBHLG, where in the former, only only gluon radiation from the initial state is treated. Sample Monte Carlo data from SSCBHLG are illustrated in the next section. Similar data for SSCYFSG will appear elsewhere [5].

We should stress that two limits of the QCD coupling constant are needed in (12), (13) and (14). One is the perturbative $-q^2 \gg \Lambda^2_{\rm QCD}$ regime in the hard gluon residuals $\bar{\beta}_n$ in (14). This regime is well–known and we use the standard type of formula [9]

$$\alpha_s(\mu) = \frac{12\pi}{(33-2n_f)\ln(\mu^2/(\Lambda^{\overline{MS}}_{n_f})^2)} \tag{19}$$

with n_f =number of open flavors and $\Lambda^{\overline{MS}}_n$ the respective value of $\Lambda_{\rm QCD}$: we take $\Lambda^{\overline{MS}}_4 \simeq$ .238 GeV for definiteness.

The second regime occurs in $\rm SUM_{IR}$ in (9), where the $k^2 \to 0$ limit is relevant for "on–shell" gluons of 4–momentum k. (We follow Tarrah [10] and use the concept of on–shell quarks and gluons in perturbation theory at large momentum transfer.) Recently [11], it has been pointed out by Mattingly and Stevenson that this limit of α_s/π exists as an infrared fixed point and its limiting value is $\simeq .263$. We use this value of (α_s/π) in $\rm SUM_{IR}$ in (9) and (13). Because of the fixed point behavior, the value one uses for the $k^2 \to 0$ coupling of gluons is not sensitive to the precise value $\Lambda^{\overline{MS}}_4$, for example. Further, since we only work to $O(\alpha_s)$ in the hard interactions, there is no contradiction between our use of $\Lambda^{\overline{MS}}$ values in (19) and the renormalization scheme of Mattingly and Stevenson in Ref. [11].

We should also note that our light quark masses are always those determined by Leutwyler and Gaisser in Ref. [12]: $m_u(1\ {\rm GeV}) \simeq 5.1 \times 10^{-3}$ GeV, $m_d(1\ {\rm GeV}) \simeq 8.9 \times 10^{-3}$ GeV, $m_s(1\ {\rm GeV}) \simeq .175$ GeV, $m_c(1\ {\rm GeV}) \simeq 1.3$ GeV, $m_b(m_b) \simeq 4.5$ GeV; for m_t, we take $m_t(m_t) \simeq$ 164 GeV. The running of these masses is readily incorporated into our calculations [12]; we ignore this running in the current paper, since it does not affect our results at the level of accuracy of interest to us here.

With these explanatory remarks in mind, we now turn to explicit Monte Carlo data illustrations for multiple gluon effects in $q + {}^{(}\bar{q}^{)\prime} \to q'' + {}^{(}\bar{q}^{)\prime\prime\prime} + n(G)$ at SSC/LHC energies in the ATLAS-CMS(SDC-GEM) acceptances.

3 Results

In this Section, we illustrate our multiple–gluon Monte Carlo event generators in the sample case of $u+u \to u+u+n(G)$, where we require the out–going u quarks to satisfy $|\eta| \le 2.8$ and $E > 58$ GeV, for definiteness, at $\sqrt{s} = 15.4$ TeV. (We comment about the SSC case $\sqrt{s} = 40$ TeV as well.) For completeness, we will show explicit Monte Carlo data for SSCBHLG, since in that generator initial, initial–final interference, and final state $n(G)$ radiation is simulated.

Our results are shown in Figs. 2, 3 and 4, where we show respectively the gluon multiplicity, the distribution of $v = (s-s')/s$, where s' is the squared invariant mass of the final quark–quark system, and the total gluon transverse momentum in TeV units. What we see is that there is a pronounced effect from the multi–gluon radiation at $\sqrt{s} = 15.4$ TeV: the average value of the number of radiated gluons with energy $E_G > 3$ GeV is, e.g., at $\sqrt{s} = 15.4$ TeV,

$$\langle n_G \rangle = 28.5 \pm 5.5. \tag{20}$$

The average value of v is, for $\sqrt{s} = 15.4$ TeV,

$$\langle v \rangle = 0.92 \pm 0.14. \tag{21}$$

The average value of the total transverse momentum in gluons is

$$\langle p_{\perp,tot} \rangle \equiv \langle |\sum_{i=1}^{n} \vec{k}_{i\perp}| \rangle = (0.19 \pm 0.23)\ \text{TeV}. \tag{22}$$

This means that a substantial amount of incoming and outgoing energy is radiated into gluons (due to the usual collinearity of radiation with its source, most of the final state radiation would be a part of the typical jet associated with its parent quark). Entirely similar results hold for the SSC $\sqrt{s} = 40$ TeV case. Evidently, any realistic treatment of QCD corrections to corrections to LHC/SSC processes must analyze the full event–by–event n–gluon effects as they interact with the detector efficiencies and cuts. Our SSCBHLG event generator provides the first amplitude–based exponentiated Monte Carlo realization of such n–gluon effects and we hope to participate in a study of their interplay with detector effects elsewhere [5].

An important consequence of our work is a prediction for the effect on the overall normalization of an LHC/SSC physics process due to multiple–gluon radiation, in an unambiguous

way, since all IR–singular effects have canceled out of our calculation. For example, if we simply compute the $O(\alpha_s)$ soft cross section, $E_G < 3$ GeV, we get (for $u + u \to u + u + (G)$, $\sqrt{s} = 40$ TeV) that

$$\sigma/\sigma_{Born} = 1 + \delta_{vir} + \delta_{real} = -22.1, \tag{23}$$

whereas if we compute the cross section for $u+u \to u+u+n(G)$ and require $v < 3$ GeV/7.7 TeV= .00039, we get

$$\sigma/\sigma_{Born} = 1.2 \times 10^{-11}. \tag{24}$$

Hence, we see that multi–gluon radiation is crucial to getting the proper normalization of the cross section.

We emphasize that the results for $\sqrt{s} = 40$ TeV give similar conclusions [5] to those in Figs. 2–4. Entirely analogous results hold for the pure initial–state multigluon event generator SSCYFSG, except only two lines radiate, so that $\langle n_G \rangle$ is reduced by a factor ~ 2, with the appropriate corresponding reductions in $\langle v \rangle$ and $\langle p_{\perp,tot} \rangle$. Such results are effectively an integration over the events used in Figs. 2–4, wherein one integrates over all gluons within some cones close to the outgoing fermions. We do not show a separate set of plots for SSCYFSG here but we will present a more detailed discussion of the respective initial–state n–gluon radiation elsewhere [5]. Here, we are focusing on the complete gluon radiation problem.

4 Conclusions

In this paper, we have shown that the IR singularities in fermion–(anti–)fermion scattering at LHC/SSC energies in QCD cancel at order α_s, permitting an immediate extension of YFS QED exponentiation methods to such processes. The YFS Monte Carlo methods invented by two of us [14] (S.J. and B.F.L.W.) for QED radiation in Z^0 physics can then be extended to multi–gluon Monte Carlo event generators with well–defined IR behavior to all orders in α_s. Such a Monte Carlo realization of amplitude–based n–gluon effects opens the way to a new era in higher–order QCD corrections to hard processes such as those of interest to the LHC/SSC physics program.

More specifically, we have realized our exponentiated, n–gluon effects via the Monte Carlo event generators SSCBHLG and SSCYFSG. Since the latter amounts to an initial–state restriction of the former, we have presented results from the former generator in this paper. We find, for example, that the average number of radiated gluons in $u + u \to u + u + n(G)$ at $\sqrt{s} = 15.4$ TeV is 28.5 ± 5.5 and that $\langle v \rangle = 0.92 \pm 0.14$. Further, the cross–section normalization is strongly affected by our n–gluon effects. Hence, only by taking these effects into account, as they interact with detector simulation, for example, can a truly detailed picture of the SSC/LHC physics platform be obtained. We hope to participate in such an investigation elsewhere.

In summary, we have presented, for the first time, the realistic event–by–event Monte Carlo realization of multi–gluon effects in hard QCD processes at SSC/LHC energies in which IR singularities are canceled to all orders in α_s. A new approach to LHC/SSC physics has thus been created. We look forward with excitement to its many applications!

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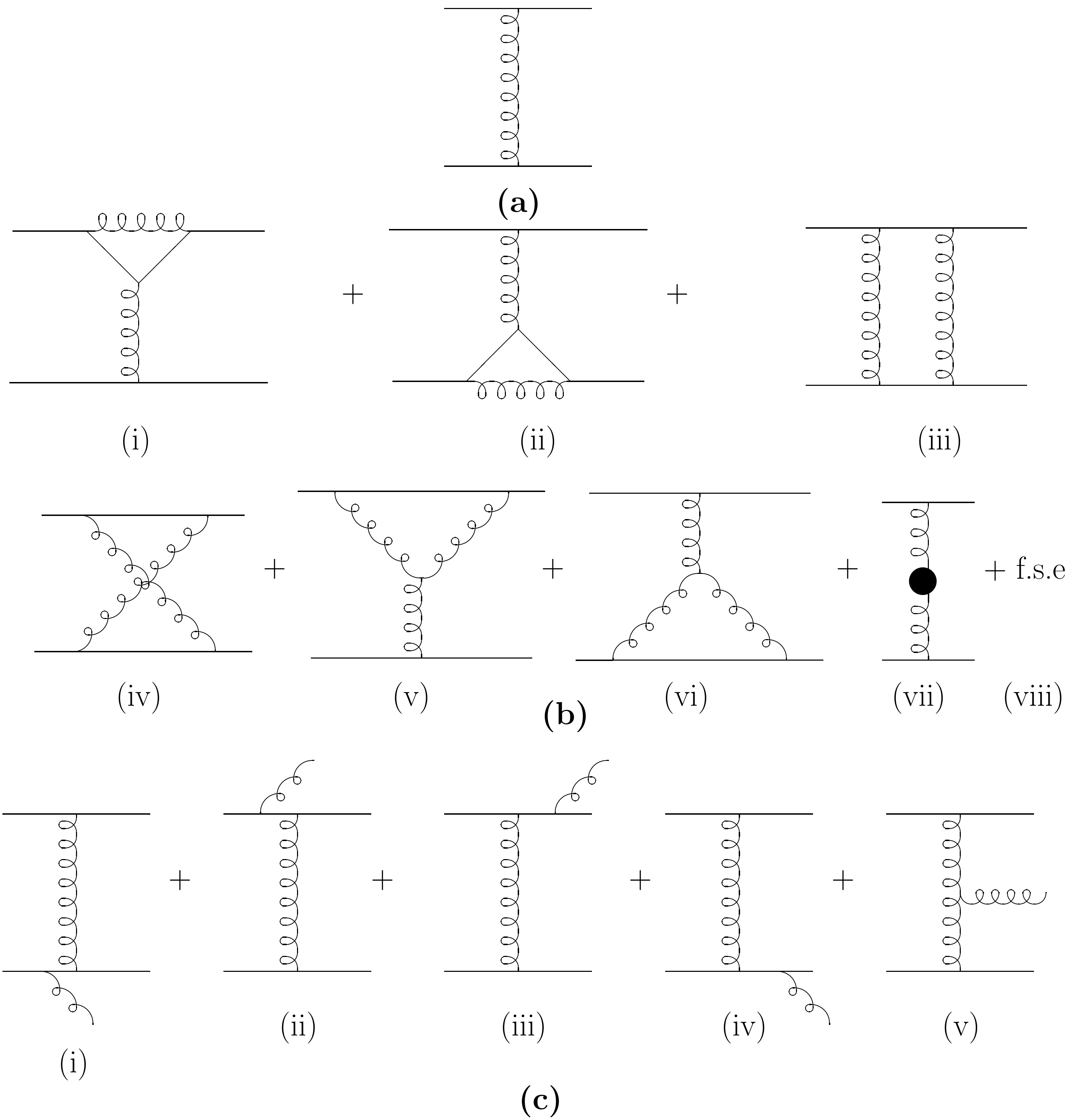

Figure 1: The process $q + {}^{(}\bar{q}^{)\prime} \to q + {}^{(}\bar{q}^{)\prime} + (G)$ to $O(\alpha_s)$: (a) Born approximation; (b) $O(\alpha_s)$ virtual correction; (c) $O(\alpha_s)$ bremsstrahlung process. Here, f.s.e. represents the fermion self-energies.

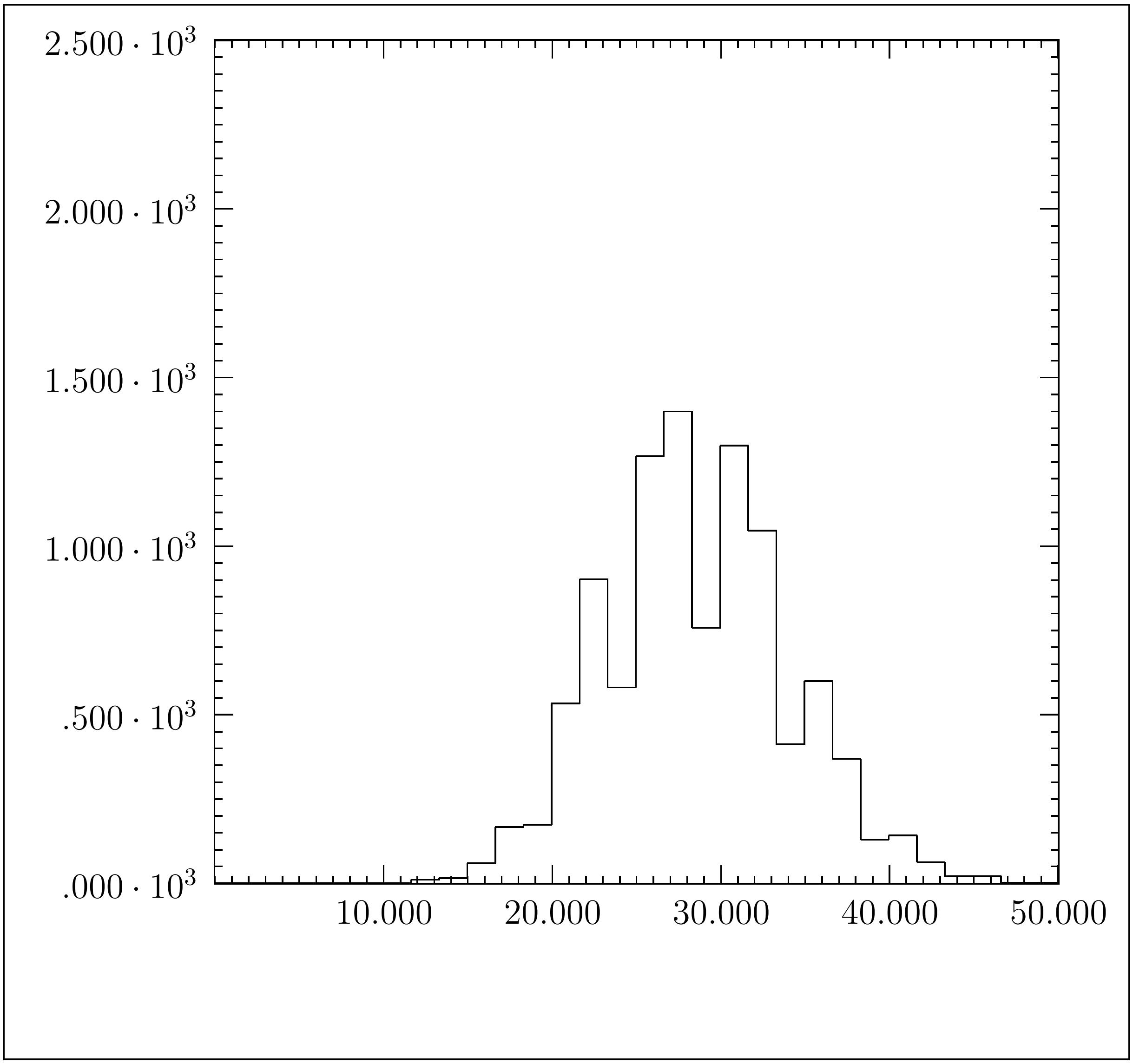

Figure 2: Gluon multiplicity distribution in $u + u \to u + u + n(G)$ at $\sqrt{s} = 15.4$ TeV.

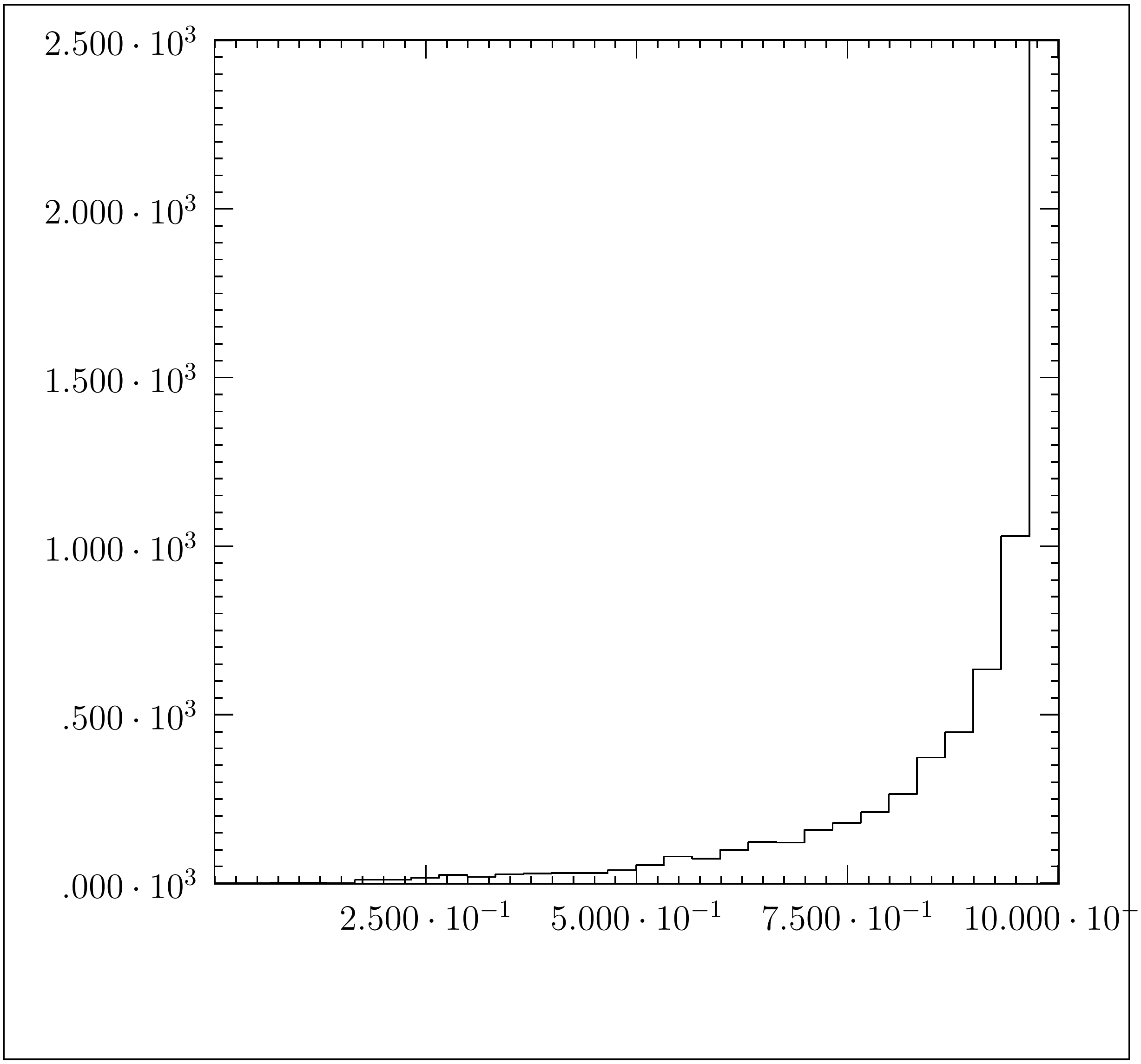

Figure 3: v–distribution in $u + u \to u + u + n(G)$ at $\sqrt{s} = 15.4$ TeV.

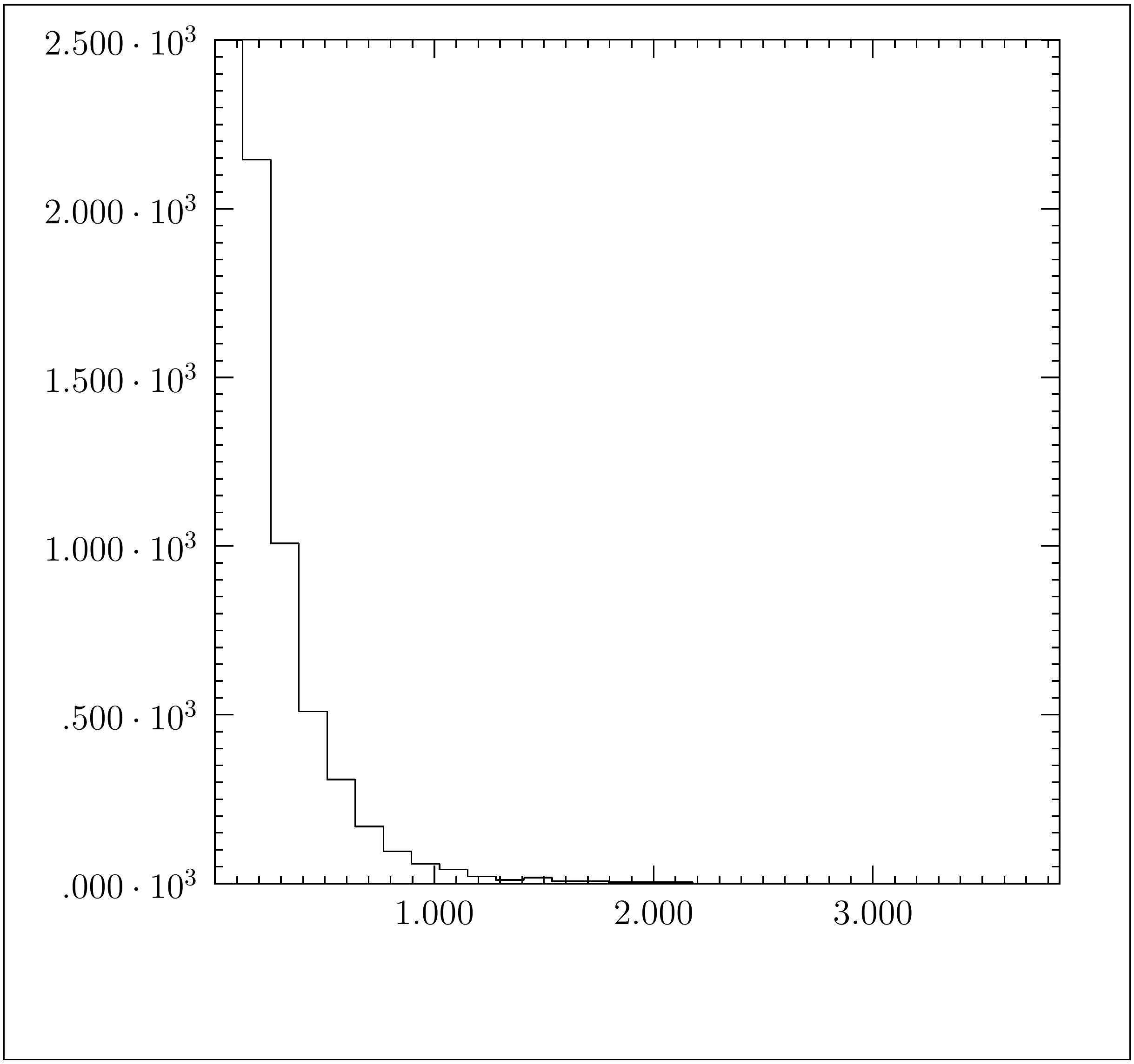

Figure 4: Total transverse momentum of gluons in $u + u \rightarrow u + u + n(G)$ at $\sqrt{s} = 15.4$ TeV. The units of the horizontal axis are TeV.